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Atmospheric Research 36 (1995) 243–249

ATMOSPHERIC
RESEARCH

Simulation of icicle growth using a three-dimensional random walk model

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Received 18 October 1993; accepted 22 February 1994

Abstract

A discrete three-dimensional random walk model has been developed to simulate icicle growth. Water flow along the icicle's surface is divided into fluid elements which follow a stochastic path towards the icicle tip. During its motion, a fluid element may freeze on the icicle's lateral surface or at its tip. The fluid elements may also drip from the icicle tip. The influence of the model parameters (the freezing probability, the shedding parameter and the motion parameter) on the icicle geometry and on the efficiency of freezing has been examined. The model predicts a characteristic three-dimensional icicle surface of a stochastic nature. This new model may be used to simulate icicle formation on transmission lines, offshore structures and buildings. A distinct advantage of the random walk model is that it is not limited by the object's geometry.

1. Introduction

Numerical models of fresh water icicles (Makkonen 1988, Johnson and Lozowski 1988) and brine icicles (Chung and Lozowski 1990) have been developed recently. These models are based on numerical solutions of continuous conservation equations. Recently, Szilder and Lozowski (1993b) formulated an analytical, time-dependent model of icicle growth based on the differential forms of the conservation of energy and mass. The analytical solution of those equations provides interesting insights into icicle growth, but the model cannot handle significant departures from a simple geometrical form.

The random walk method was introduced into the modelling of the wet ice formation process by Szilder (1993). Szilder and Lozowski (1993a) have used this stochastic approach to simulate icicle growth. A two-dimensional model, with cylindrical symmetry around the vertical axis, was used to simulate the three-dimensional process. Fluid elements were released at the icicle's root and were allowed to follow irregular paths downwards

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SSDI 0169-8095(94)00039-5

within a modified two-dimensional lattice. The model parameters were expressed as functions of the heat transfer to the airstream and the water supply rate, using simple analytical considerations.

In this paper, a fully three-dimensional simulation of icicle growth is presented. A detailed analysis of the influence of the random walk model parameters on structure, shape and the freezing efficiency is undertaken. However, specific relations between model parameters and physical parameters are not derived. The derivation of such relations, along with model verification through experiments, is the subject of a continuing research program.

2. Model description

The behaviour of water flowing along an icicle surface is not necessarily a continuous process. Frequently the water is seen to form beads which follow circuitous channels as they make their way around roughness elements on the surface. In the present work, this behaviour is modelled by the random walk of fluid elements in a three-dimensional lattice. The water mass flux is divided into fluid elements which are released at the icicle root and move randomly along the existing icicle surface. A fluid element may freeze on the icicle's lateral surface or at the tip, or it may drip from the tip. At each step, a random number is generated to determine the next move of the fluid element, according to a pre-determined distribution of motion and freezing probabilities. The fluid element may move only to an adjacent empty cell where it will have at least one neighbour. A neighbour is defined as a frozen ice element at any of 26 neighbouring cells. If a random number resulting in motion into a cell without a neighbour is chosen, the fluid element remains temporarily at its current location, and a new random number is generated to determine the motion. This condition prevents the fluid element from "walking away" from the icicle's surface. More details of the random walk approach can be found in Szilder (1993) and Szilder and Lozowski (1993a).

The model has four parameters: the motion parameter (which determines the relation among the probabilities of motion), the freezing probability (which determines the likelihood of freezing at any location), the shedding parameter (which determines the number of time steps allowed for an element to remain unfrozen at the icicle tip before dripping) and the freezing range (which provides the side of the cube within which the final location of the element is sought after a freezing decision has been made). All four model parameters are discussed in detail in the following sections.

The random walk of a fluid element consists of a series of moves through the three-dimensional lattice of Fig. 1. At each time step there are seven possible outcomes for the fluid element: it may move one cell in any of six perpendicular directions, or it may freeze "in situ". For simplicity, it has been assumed that the probabilities of moving horizontally (in four perpendicular directions) and upwards are the same. The ratio of the probability of moving downwards to the probability of motion in any other direction is defined as the motion parameter. This parameter and the probability of freezing determine the probabilities of motion in all six directions. It is conjectured that the value of the motion parameter is associated with the relative magnitude of gravity, surface tension and viscous forces. Fur-

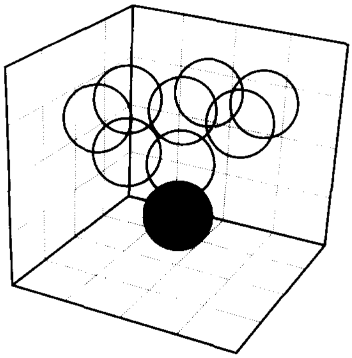

Fig. 1. A schematic representation of the three-dimensional model. The moving fluid element is represented by the solid sphere and already frozen elements by the open spheres.

thermore, the freezing probability is undoubtedly related to the heat exchange conditions between the icicle surface and the surrounding air.

If a fluid element moves along the icicle's surface without freezing, it will eventually reach the icicle's tip. If it remains at the tip without freezing for the number of time steps specified by the shedding parameter, it drips and leaves the ice structure. Physically, this parameter is related to the supply rate. An increase of the supply rate leads to faster growth of the pendant drop and more frequent dripping from the tip. This is modelled by decreasing the shedding parameter.

When a random number corresponding to the freezing probability is chosen, a "cradle" location is sought for the fluid element in the immediate neighbourhood of its current location. The size of this neighbourhood is determined by the freezing range parameter. The neighbourhood is a cube, centered on the current location of the fluid element, with a side length in cells equal to double the freezing range parameter plus one. The fluid element is moved to the empty cell within this volume where it will have the maximum number of neighbours. If there is more than one such location, the final site is chosen randomly from among them. This process simulates qualitatively the effect of surface tension forces which tend to minimize local surface curvature.

All the results presented in the next section have been obtained by considering a three-dimensional lattice of 40 by 40 by 80 cells (the latter in the vertical direction). The size of the domain was chosen as a compromise between model resolution and running time of the program. Each simulation was started with a single seed particle located at the top centre of the lattice. The fluid elements were released consecutively from below the top surface of the model domain towards the forming icicle. The simulation was terminated when the growing structure reached any boundary of the domain. Consequently, the number of released fluid elements was not fixed from simulation to simulation.

3. Results

The icicle's shape is determined by the four model parameters. The results of the simulations reveal that the freezing range parameter controls the smoothness of the icicle's

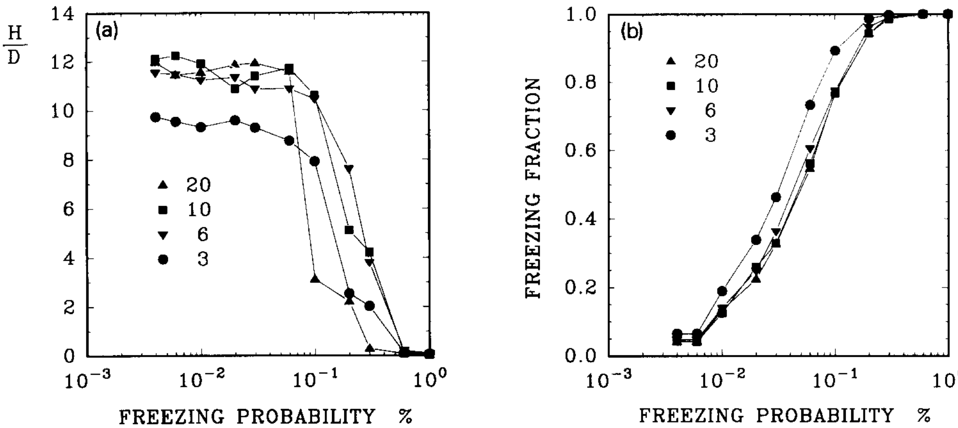

Fig. 2. The influence of the freezing probability on the icicle formation process. The different curves are obtained for four values of the motion parameter, 3, 6, 10 and 20. The shedding parameter is 1000. (a) the ratio of icicle length to its average diameter. (b) the fraction of the delivered water which forms the icicle.

surface. When this parameter is zero (a ''cradle'' location is not sought), a very incoherent and rime-like porous structure is obtained. If this parameter is unity, a well-pronounced icicle surface is obtained and the voids inside the structure disappear. For still higher values of the freezing parameter, the surface is very smooth and the icicle becomes thick. Consequently, we assume here that the freezing range parameter is two, and the following results have been obtained using this value.

Let us first examine the influence of the freezing probability on the predicted structure, assuming the shedding parameter to be 1000, and using four values of the motion parameter, 3, 6, 10 and 20 (Fig. 2). Two parameters have been employed to describe the resulting icicle: the ratio of the icicle's length to its average diameter (the average diameter is defined as the diameter of a cylinder containing all the frozen fluid elements and having the same length as the icicle), and the freezing fraction (the ratio of the number of frozen fluid elements to the total number of elements delivered to the icicle root). When the freezing probability is smaller than 0.004%, an icicle is not formed. Very small values of the freezing probability, for the specified value of the shedding parameter, prevent the fluid elements from freezing and all the elements drip from below the seed.

For higher values of the freezing probability, the initial freezing around the seed allows growth to continue until the icicle length reaches a maximum extent (predefined by the vertical domain size). Thin icicles are formed (Fig. 2a) and substantial dripping occurs (Fig. 2b). An icicle shape representative of this range of the freezing probability is shown in Fig. 3a. An irregular icicle surface can be observed. The irregular surface is a natural and realistic consequence of this type of modelling. The spiky fine scale structure is caused by the combination of coarse resolution and the rendering program used to produce the surface images. It is not entirely realistic, as it stands, but this could be improved with higher resolution. An increase in the freezing probability by an order of magnitude from 0.004 to 0.060% does not significantly change the final shape of the icicle. However, the percentage of water dripping from the icicle tip decreases substantially (Fig. 2b). In this range of the freezing probability, the motion parameter has a small influence on the icicle growth. Since

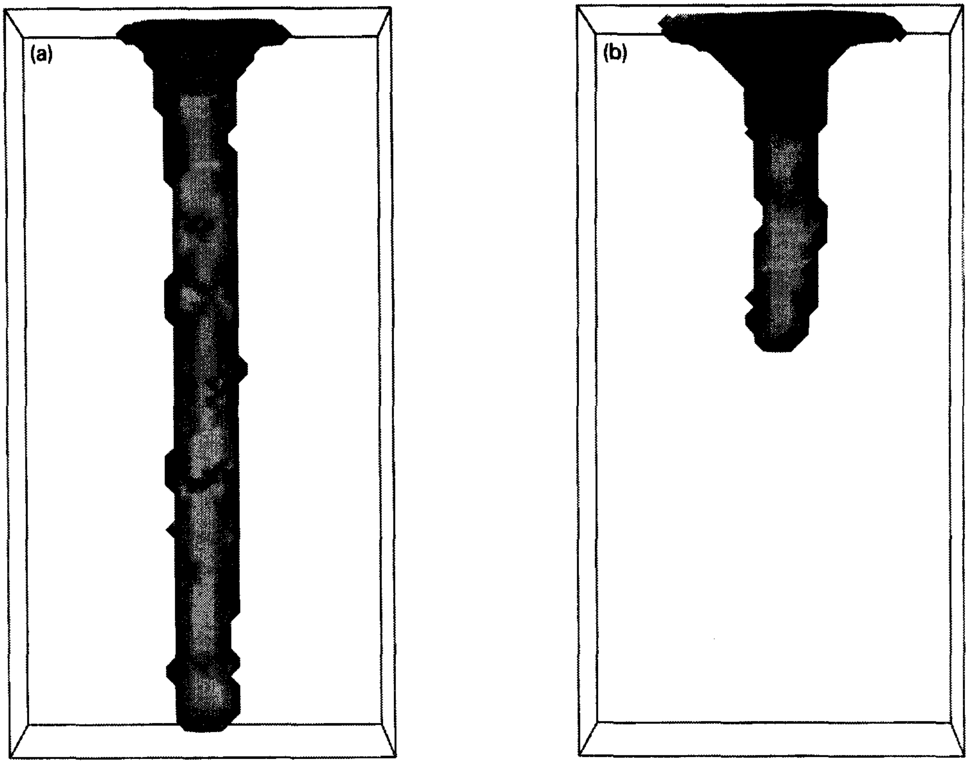

Fig. 3. Icicle shape prediction for a shedding parameter of 1000 and a motion parameter of 6. The lattice size is 40 by 40 by 80 cells. (a) freezing probability of 0.01% and total number of delivered fluid elements of 18,750 give freezing fraction of 0.140. (b) freezing probability 0.30% and total number of delivered fluid elements of 2150 give freezing fraction of 0.990.

a smaller value of the motion parameter is associated with a smaller probability of downward motion, a smaller number of fluid elements reach the icicle's tip and less dripping occurs. Consequently, the icicle is thicker (Fig. 2a) and the freezing fraction is higher (Fig. 2b). Within a moderate range of the motion parameter (from about 6 to 20), its magnitude does not substantially influence the model's predictions.

When the freezing probability is higher than 0.06%, an increase in the number of fluid elements freezing on the icicle's lateral surface substantially changes the availability of fluid elements at the icicle's tip and the icicles grow thicker and shorter (Fig. 3b). A further increase of the freezing probability leads to the freezing of all fluid elements just under the top surface of the domain. The magnitude of the motion parameter has an interesting influence on the ratio of the icicle's length to its diameter. An optimum value of the motion parameter for maximum vertical elongation seems to exist. Both smaller and larger values of the motion parameter tend to make the icicles shorter. For an intermediate value, the effectiveness of reaching the lower portions of the icicle is the highest.

The magnitude of the shedding parameter influences the icicle's structure (Fig. 4). Small values of this parameter mean that for a given probability of freezing, the likelihood of the

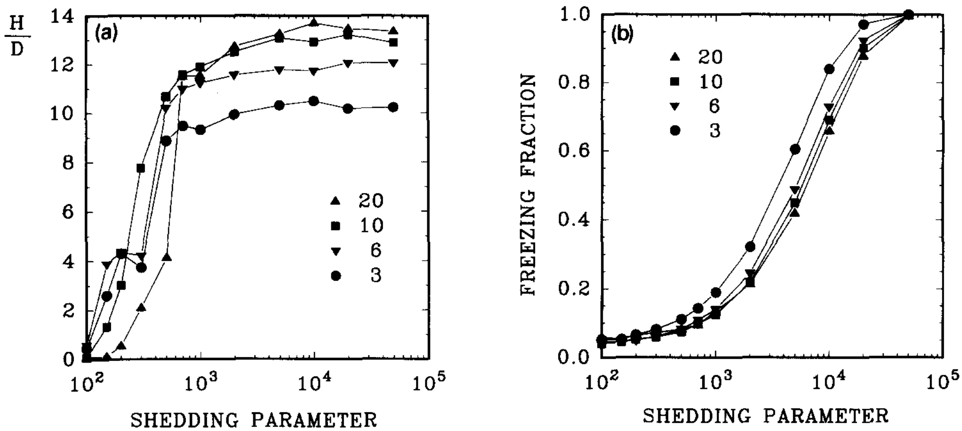

Fig. 4. The influence of the shedding parameter on the icicle formation process. The different curves are obtained for four values of the motion parameter, 3, 6, 10 and 20. The freezing probability is 0.01%. (a) the ratio of icicle length to its average diameter. (b) the fraction of the delivered water which forms the icicle.

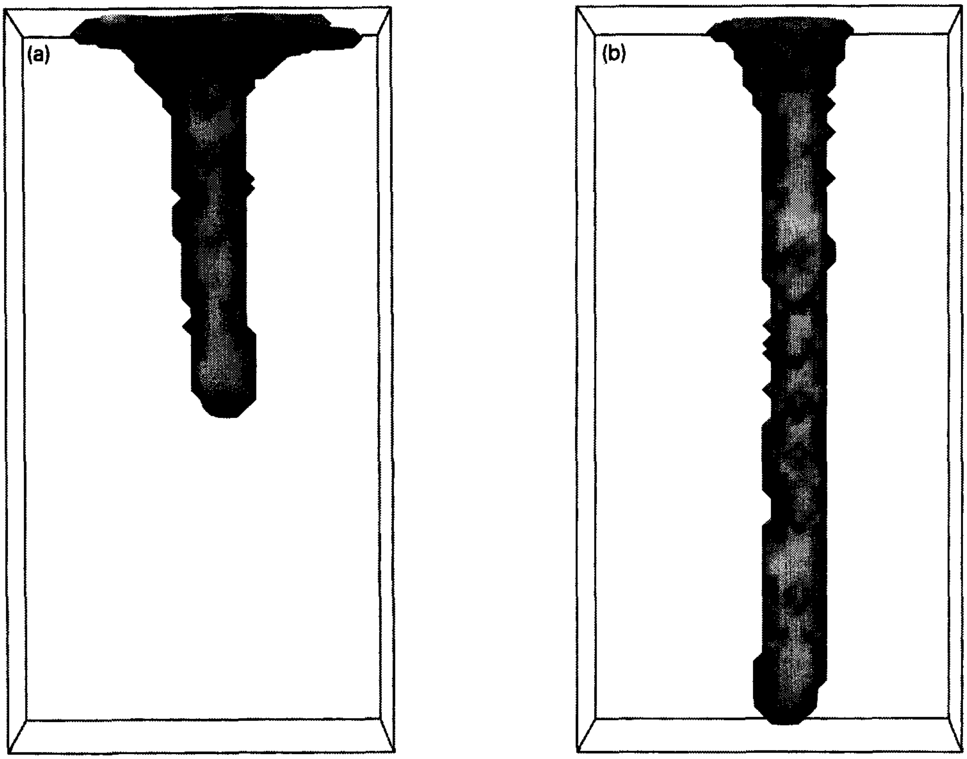

Fig. 5. The icicle shape prediction for a freezing probability of 0.01% and a motion parameter of 6. The lattice size is 40 by 40 by 80 cells. (a) shedding parameter 200 and total number of delivered fluid elements of 57,500 give freezing fraction of 0.050. (b) shedding parameter 20,000 and total number of delivered fluid elements of 2450 give freezing fraction of 0.923.

fluid element freezing at the icicle's tip is small. Consequently, an increase of the shedding parameter leads to the formation of longer icicles (Fig. 4a) and a larger fraction of the delivered water freezes (Fig. 4b). When the shedding parameter is smaller than approximately 1000, the motion parameter has a complex influence on the icicle's geometry (Fig. 4a). Both very small and very large values of the motion parameter restrict the number of fluid elements reaching the icicle's tip. An example of the icicle structure in this range of the shedding parameter is shown in Fig. 5a. When the shedding parameter exceeds about 1000, the final icicle geometry remains unchanged (compare Fig. 3a with Fig. 5b) and only the freezing fraction increases with the shedding parameter (Fig. 4b).

4. Conclusions

A new random walk model predicts three-dimensional icicle structures. The influence of the model parameters on the icicle geometry has been investigated. A decrease of the freezing probability, which is related to heat loss from the icicle, makes the icicles longer and more water drips from the tip. For very small values of the freezing probability however, the formation of icicles ceases. A decrease of the shedding parameter, which is inversely related to the supply rate and rate of dripping, causes the icicles to shorten. The magnitude of the motion parameter has a limited influence on icicle geometry. The fluid elements travel in an efficient way downwards along the icicle surface for intermediate values of the motion parameter. In the future, the relation between the model and the physical parameters will be established and verified using icicle growth experiments. The three-dimensional random walk model described here allows the simulation of ice formation due to gravitational flow of liquid on objects of complex geometry.

Acknowledgements

This research was financially supported through an NSERC research grant.

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